Regional Mathematical Olympiad 2026

Questions and Worked Solutions

STUDENT EDITION

Six questions, with two solutions to each.

Read a question and try it before turning to its solutions. Work through one method fully; then compare it with the other to see what changes and what stays the same.

The two approaches

Question	First solution	Second solution
1 · Geometry	Two circles and an angle chase	Coordinates and perpendicular vectors
2 · Combinatorics	An explicit alternating assignment	A different assignment built from a path
3 · Inequalities	Nonnegative squares	AM–GM and Cauchy–Schwarz
4 · Geometry	Chord products and a parallelogram	Circle equations in coordinates
5 · Number theory	Remainders modulo $a^b + 1$	The smallest possible exponent
6 · Inequalities	Pairing complementary powers	A bound obtained from AM–GM

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Class: _____ Date: _____

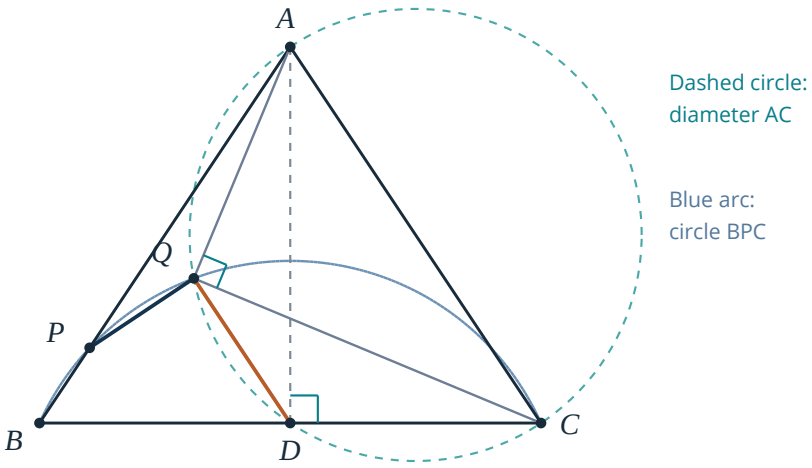
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Question 1 | Geometry

Let ABC be a triangle with AB = AC. Let D be the midpoint of BC and P be a point on the segment AB satisfying ∠CPA < 90°. Suppose Q (≠ P) is a point on the circumcircle of triangle BPC such that ∠AQC = 90°. Prove that ∠DQP = 90°.

Solution 1 | Using two circles

Draw AD, AQ, CQ, DQ and PQ. The useful observation is that the two right angles at D and Q put four points on another circle.



Write ∠ABC = ∠BCA = β. In an isosceles triangle, the median from A is also an altitude and an angle bisector. Hence

$$AD \perp BC, \qquad \angle DAC = 90^\circ - \beta.$$

Since ∠ADC = ∠AQC = 90°, the points A, D, Q and C lie on the circle with diameter AC. Angles subtending its chord DC are equal, so

$$\angle DQC = \angle DAC = 90^\circ - \beta.$$

Now use the original circle. The condition ∠CPA < 90° gives the position shown, with B, P, Q, C in that order on the circle. Opposite angles in BPQC add to 180°:

$$\angle PQC = 180^\circ - \angle PBC = 180^\circ - \beta.$$

The ray QD lies inside ∠PQC. Subtracting the smaller angle gives

$$\angle DQP = (180^\circ - \beta) - (90^\circ - \beta) = 90^\circ.$$

Question 1 | Second solution

Solution 2 | Coordinates

Place the base on the horizontal axis. We may change the scale without changing any angle, so take

$$C=(0,0),\quad B=(2,0),\quad D=(1,0),\quad A=(1,h),\quad h>0.$$

A point P inside AB can then be written as

$$P=(1+t,\ h(1-t)),\qquad 0<t<1.$$

Find the equations satisfied by Q.

A circle through C and B has equation $x^2 + y^2 - 2x + ky = 0$. Substituting P determines k. It is convenient to put $m = h + k$:

$$k=\frac{1+t}{h}-h(1-t),\qquad hm=1+(1+h^2)t.$$

Let Q = (r, s). Because AQ is perpendicular to CQ, the products of the corresponding vector components add to zero:

$$(1-r)(-r)+(h-s)(-s)=0,\qquad r^2+s^2=r+hs.$$

The equation of the original circle also gives $r^2 + s^2 = 2r - ks$. Comparing the two expressions,

$$r=(h+k)s=ms.$$

Q is not C, so $s \neq 0$. Substitution in $r^2 + s^2 = r + hs$ now gives

$$(1+m^2)s=m+h,\qquad s=\frac{m+h}{1+m^2},\quad r=\frac{m(m+h)}{1+m^2}.$$

Check the required perpendicularity.

The vectors QD and QP are perpendicular if their component products add to zero. Using $r^2 + s^2 = r + hs$, that sum is

$$(1-r)(1+t-r)-s\,[h(1-t)-s]$$

$$=r^2+s^2-(2+t)r-h(1-t)s+1+t$$

$$=(1+t)(1-r)+hts$$

$$=\frac{(1+t)(1+m^2)-(1+t)m(m+h)+ht(m+h)}{1+m^2}$$

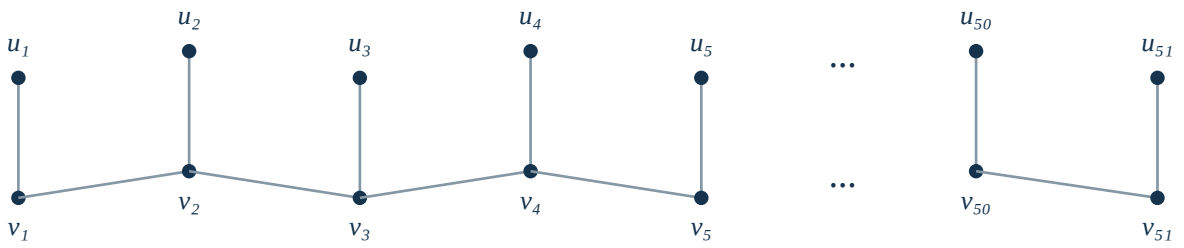
$$=\frac{1+t-hm+h^2t}{1+m^2}=0.$$

The last numerator is zero because $hm = 1 + (1 + h^2)t$. Therefore

$$QD\perp QP,\qquad \angle DQP=90^\circ.$$

Question 2 | Combinatorics

Consider the following diagram with 102 vertices.



Show that one can assign the numbers 1, 2, …, 102 to the 102 vertices u_1, u_2, …, u_{51}, v_1, v_2, …, v_{51} such that no two vertices have the same number, and the absolute differences of the numbers on adjacent vertices are a rearrangement of 1, 2, …, 101. Two points are adjacent if a line segment joins them in the diagram.

Solution 1 | An alternating assignment

Put the small and large numbers alternately along the lower row. In each column i, use the pair i and 103 − i:

i	1	2	3	4	5	…	50	51
u_i	102	2	100	4	98	…	50	52
v_i	1	101	3	99	5	…	53	51

More precisely, if i is odd, put i at v_i and 103 − i at u_i. If i is even, reverse these two labels. The column pairs use 1 to 51 and 52 to 102, so every number appears exactly once.

Vertical edges

$$|u_i - v_i| = 103 - 2i \qquad (1 \le i \le 51).$$

These are 101, 99, 97, …, 3, 1: all the odd differences.

Edges along the lower row

$$|v_{i+1} - v_i| = 102 - 2i \qquad (1 \le i \le 50).$$

These are 100, 98, 96, …, 4, 2: all the even differences.

Together, the edges give every difference from 1 to 101 exactly once.

Question 2 | Second solution

Solution 2 | Build the assignment from a path

We first arrange the numbers 1 to 51 in a row so that neighbouring numbers have all the differences 1 to 50. This row will produce a different assignment of the 102 labels.

Make an auxiliary row.

Start with the middle numbers 14 to 38, taking the smallest, then the largest, and continuing inwards:

$$14,\ 38,\ 15,\ 37,\ \ldots,\ 25,\ 27,\ 26.$$

The differences in this block are 24, 23, …, 1. Follow it by the remaining numbers, alternating between the two ends:

$$1,\ 51,\ 2,\ 50,\ \ldots,\ 13,\ 39.$$

The joining difference is $|26 - 1| = 25$. The differences within the second block are 50, 49, …, 26. Thus the complete row has each difference 1 to 50 exactly once.

Turn this row into vertex labels.

Call the successive numbers $t_1, t_2, \ldots, t_{51}$, and put

$$v_i = 2t_i - 1, \qquad u_i = 104 - 2t_i.$$

The lower labels are all the odd numbers 1 to 101; the upper labels are all the even numbers 2 to 102. No label is repeated. For example, the assignment begins

i	1	2	3	4
t_i	14	38	15	37
v_i	27	75	29	73
u_i	76	28	74	30

Check both kinds of edge.

$$|v_{i+1} - v_i| = 2|t_{i+1} - t_i|.$$

So the lower-row differences are exactly 2, 4, …, 100. A vertical difference is

$$|u_i - v_i| = |105 - 4t_i|.$$

For $t_i = 1, \ldots, 26$, these values are 101, 97, …, 1. For $t_i = 27, \ldots, 51$, they are 3, 7, …, 99. These two lists contain all the odd numbers 1 to 101, without repetition.

Hence this second assignment also gives precisely the required 101 differences.

Question 3 | Inequalities

Prove that for any two real numbers x and y, we have

$$(x^6 - x^2 + 3)(y^6 - y^2 + 3) > 2(x^2 + y^2)^2.$$

Solution 1 | Completing squares

Put u = x² and v = y². Both are nonnegative. We shall compare each cubic expression with a quadratic one.

$$u^3 - u + 3 = u^2 + 2 + (u-1)^2(u+1).$$

The last term is nonnegative, so

$$u^3 - u + 3 \geq u^2 + 2,$$

with equality only at u = 1. In the same way,

$$v^3 - v + 3 \geq v^2 + 2,$$

with equality only at v = 1. Both lower bounds are positive; multiplying them preserves the inequality:

$$(u^3 - u + 3)(v^3 - v + 3) \geq (u^2 + 2)(v^2 + 2).$$

Now complete one more square:

$$(u^2 + 2)(v^2 + 2) = 2(u+v)^2 + (uv - 2)^2.$$

$$(u^3 - u + 3)(v^3 - v + 3) \geq 2(u+v)^2.$$

For equality, the first comparison would require u = v = 1, while the last square would require uv = 2. These conditions cannot hold together. The inequality is therefore strict.

$$(x^6 - x^2 + 3)(y^6 - y^2 + 3) > 2(x^2 + y^2)^2.$$

Question 3 | Second solution

Solution 2 | AM–GM and Cauchy–Schwarz

Again write $u = x^2$ and $v = y^2$. This time we obtain the needed bounds from familiar inequalities, without expanding the difference of the two sides.

Apply AM–GM twice.

For any $t \geq 0$, AM–GM applied to $t^3, t^3, 1$ and then to $t^3, 1, 1$ gives

$$2t^3 + 1 \geq 3t^2, \qquad t^3 + 2 \geq 3t.$$

Adding and dividing by 3,

$$t^3 + 1 \geq t^2 + t.$$

Consequently,

$$t^3 - t + 3 \geq t^2 + 2,$$

with equality only when $t = 1$. Use this once with $t = u$ and once with $t = v$:

$$(u^3 - u + 3)(v^3 - v + 3) \geq (u^2 + 2)(v^2 + 2).$$

Apply Cauchy–Schwarz.

Use the pairs $(u, \sqrt{2})$ and $(\sqrt{2}, v)$. Cauchy–Schwarz gives

$$(u^2 + 2)(2 + v^2) \geq (u\sqrt{2} + \sqrt{2}\,v)^2.$$

$$(u^3 - u + 3)(v^3 - v + 3) \geq 2(u + v)^2.$$

Equality in the AM–GM bounds requires $u = v = 1$. Equality in Cauchy–Schwarz requires the two pairs to be proportional:

$$\frac{u}{\sqrt{2}} = \frac{\sqrt{2}}{v}, \qquad uv = 2.$$

That is incompatible with $u = v = 1$. Thus equality is impossible, and

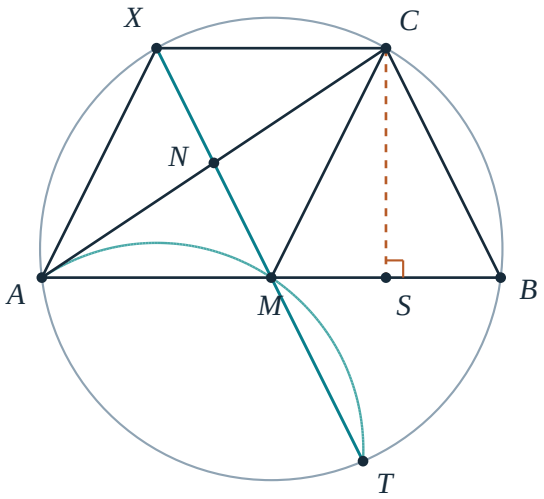
$$(x^6 - x^2 + 3)(y^6 - y^2 + 3) > 2(x^2 + y^2)^2.$$

Question 4 | Geometry

Let ABC be an acute triangle and let M and N be the midpoints of AB and AC, respectively. Let T be the point of intersection of ray NM with the circumcircle of ABC. Suppose that the circumcircle of ATM is tangent to AC at A. Let S be the foot of the perpendicular from C on AB. Determine the value of AS/SB.

Solution 1 | Chord products and a parallelogram

Extend MN through N to meet the circle ABC at X. The order on this line is X, N, M, T.



X is the other point on line NM and the circumcircle of ABC.

CS is perpendicular to AB.

For circle ATM, NA is a tangent and NMT is a secant. Therefore

$$NA^2 = NM \cdot NT.$$

For circle ABC, chords AC and XT intersect at N. Since NA = NC,

$$NA^2 = NA \cdot NC = NX \cdot NT.$$

Comparing the two products gives NX = NM. Hence N is the midpoint of both AC and XM, and AMCX is a parallelogram. It follows that

$$CX \parallel AB, \qquad AX = MC.$$

As CX ∥ AB, we have ∠XCA = ∠BAC. These equal angles in the same circle subtend equal chords, so AX = BC. Thus MC = BC.

Triangle MCB is isosceles. Its altitude CS therefore bisects MB:

$$MS = SB = \frac{MB}{2} = \frac{AB}{4}, \qquad AM = \frac{AB}{2}.$$

$$\frac{AS}{SB} = \frac{AM + MS}{SB} = \frac{3AB/4}{AB/4} = 3.$$

Question 4 | Second solution

Solution 2 | Equations of the two circles

Take the midpoint M as the origin, with AB horizontal. Choose the scale so that

$$A=(-1,0),\quad B=(1,0),\quad M=(0,0),\quad C=(u,v),\quad v>0.$$

Then N = ((u − 1)/2, v/2). Every point on the line MN has the form λ(u − 1, v), so write

$$T=(\lambda(u-1),\lambda v),\qquad R=(u-1)^2+v^2.$$

Here λ ≠ 0 because T is not M.

Use the tangency condition.

The centre O of circle ATM lies on the perpendicular bisector of AM, so O = (−1/2, w). Its radius OA is perpendicular to AC. Taking the products of the components gives

$$\frac{u+1}{2}+wv=0,\qquad w=-\frac{u+1}{2v}.$$

As this circle passes through M = (0, 0), its equation is

$$x^2+y^2+x+\frac{u+1}{v}y=0.$$

Substitute the coordinates of T and divide by λ:

$$\lambda^2R+2u\lambda=0,\qquad \lambda R=-2u.\qquad (1)$$

Use the circumcircle of ABC.

A circle through (−1, 0) and (1, 0) has equation $x^2 + y^2 + ky = 1$. Substituting C gives

$$x^2+y^2+\frac{1-u^2-v^2}{v}y=1.$$

Substituting T, then using (1), gives

$$\lambda^2R+\lambda(1-u^2-v^2)=1,$$

$$\lambda(1-u^2-v^2-2u)=1.\qquad (2)$$

Multiply (2) by R and use λR = −2u:

$$-2u(1-u^2-v^2-2u)=(u-1)^2+v^2.$$

After collecting terms and factoring,

$$(2u-1)\big((u+1)^2+v^2\big)=0.$$

The second factor is positive, so u = 1/2. The perpendicular foot is S = (1/2, 0), and therefore

$$\boxed{\frac{AS}{SB}=\frac{1+1/2}{1-1/2}=3.}$$

Question 5 | Number theory

Let a be an integer greater than 1. Determine all positive integers b which have the property that $a^b + 1$ divides a number of the form $a^n + b$ for some positive integer n.

Solution 1 | Working with remainders

Put $m = a^b + 1$. We shall use the elementary bound

$$b \le 2^{b-1} \le a^{b-1}.$$

For $b \ge 2$, the first inequality follows by keeping the first two terms of $(1 + 1)^{b-1}$. For $b = 1$ it is an equality. Thus, whenever $0 \le r < b$,

$$0 < b + a^r \le 2a^{b-1} \le a^b < m. \qquad (1)$$

Now write $n = qb + r$, where $q \ge 0$ and $0 \le r < b$. Since $a^b = m - 1$, each factor a^b may be replaced by -1 when working modulo m. Thus

$$a^n + b \equiv (-1)^q a^r + b \pmod{m}.$$

If q is even, m would divide $b + a^r$. This is impossible by (1). If q is odd, m must divide $b - a^r$. But (1) also gives

$$|b - a^r| < m.$$

The only multiple of m with absolute value less than m is zero. Hence

$$b - a^r = 0, \qquad b = a^r.$$

Check the converse.

Let $b = a^k$, where k is any nonnegative integer. Choose $n = b + k$. Then

$$a^n + b = a^{b+k} + a^k = a^k(a^b + 1).$$

This is divisible by $a^b + 1$. Therefore all, and only, the following values work:

$$b = 1, a, a^2, a^3, \ldots.$$

Question 5 | Second solution

Solution 2 | Choose the smallest exponent

This proof uses ordinary divisibility. Suppose b works, put $m = a^b + 1$, and choose the smallest positive n for which m divides $a^n + b$.

The exponent cannot reach 2b.

If $n > 2b$, subtract the following multiple of m from $a^n + b$:

$$a^{n-2b}(a^{2b}-1) = a^{n-2b}(a^b-1)m.$$

The result is $a^{n-2b} + b$, which would also be divisible by m. Its exponent is smaller and still positive, contradicting the choice of n.

If $n = 2b$, use

$$a^{2b} + b = (a^b-1)m + (b+1).$$

Here $0 < b + 1 < m$, so this cannot be divisible by m either. Thus $n < 2b$.

The exponent cannot be smaller than b.

As in the first solution, $b \le a^{b-1}$. If $n < b$, then

$$0 < a^n + b \le a^{b-1} + b \le 2a^{b-1} \le a^b < m.$$

That is again too small to be a positive multiple of m. Therefore

$$b \le n < 2b.$$

Use this narrow range.

Write $n = b + k$, with $0 \le k < b$. Then

$$a^n + b = a^k(a^b+1) + (b-a^k) = a^k m + (b-a^k).$$

The difference $b - a^k$ must be divisible by m. Its absolute value is less than m, so it must be zero. Hence $b = a^k$.

Conversely, for $b = a^k$ the choice $n = b + k$ gives $a^n + b = a^k(a^b + 1)$. Thus

$$b = 1, a, a^2, a^3, \ldots.$$

Question 6 | Inequalities and rearrangements

Let n be a fixed positive integer and S be a fixed positive real number. Find the minimum constant c, in terms of S and n, such that:

Given any positive real numbers $a_1, a_2, \ldots, a_n$ with $a_1 + a_2 + \cdots + a_n = S$, there is a rearrangement $b_1, b_2, \ldots, b_n$ of $a_1, a_2, \ldots, a_n$ such that

$$(a_1 + b_1) + (a_2^2 + b_2^2) + \cdots + (a_n^n + b_n^n) \le c.$$

Solution 1 | Pairing complementary powers

For n = 1 the expression is 2S, so the answer is $S + S^n$. Suppose now that $n \ge 2$. Reverse the given list:

$$b_i = a_{n+1-i} \qquad (1 \le i \le n).$$

Each number a_i now occurs with exponents i and n + 1 − i. If E denotes the expression in the question,

$$E = \sum_{i=1}^{n} \left(a_i^i + a_i^{n+1-i}\right).$$

For any $t > 0$, we have the identity

$$t + t^n - t^i - t^{n+1-i} = t(t^{i-1} - 1)(t^{n-i} - 1).$$

When $t \ge 1$, both bracketed factors are nonnegative. When $0 < t \le 1$, both are nonpositive. Either way, their product is nonnegative, giving

$$t^i + t^{n+1-i} \le t + t^n.$$

Apply this to each a_i and add:

$$E \le \sum_{i=1}^{n} (a_i + a_i^n) = S + \sum_{i=1}^{n} a_i^n.$$

In the expansion of $(a_1 + \cdots + a_n)^n$, every a_i^n occurs and all other terms are nonnegative. Hence

$$\sum_{i=1}^{n} a_i^n \le S^n, \qquad E \le S + S^n.$$

This proves that $S + S^n$ always works. We still need to show that it cannot be lowered.

Question 6 | First solution, continued

Why no smaller constant works

Choose a small positive ε and put

$$t = S - (n-1)\varepsilon.$$

Use one entry equal to t and the other n − 1 entries equal to ε. As ε decreases towards zero, t approaches S. All entries remain positive.

If 0 < S ≤ 1

Put t in the first position, so $a_1 = t$. The original sum contributes at least t. In any rearrangement, t receives some exponent j with $1 \le j \le n$. Since $0 < t < 1$,

$$t^j \ge t^n.$$

Thus every rearrangement satisfies

$$E \ge t + t^n.$$

If S > 1

Choose ε small enough that $t > 1$, and put t in the last position, so $a_n = t$. The original sum contributes at least t^n. In any rearrangement,

$$t^j \ge t.$$

Once again, every rearrangement satisfies

$$E \ge t^n + t.$$

In both cases, the lower bound approaches $S + S^n$ as ε approaches zero. A proposed constant smaller than $S + S^n$ is therefore exceeded for sufficiently small positive ε, regardless of the rearrangement.

$$c_{\min} = S + S^n.$$

For n > 1 we do not set the small entries equal to zero. We only make them as small as needed. This keeps the construction within the condition that every entry is positive.

Question 6 | Second solution

Solution 2 | Compare the powers using AM–GM

The case n = 1 again gives c = 2S. Let n ≥ 2 and reverse the list, so $b_i = a_{n+1-i}$. Since $0 < a_i \le S$,

$$a_i^{\,i} \le a_i S^{i-1}, \qquad a_i^{\,n+1-i} \le a_i S^{n-i}.$$

Therefore

$$E \le \sum_{i=1}^{n} a_i (S^{i-1} + S^{n-i}).$$

Bound the two powers of S.

Apply AM–GM to n − i copies of 1 and i − 1 copies of S^{n-1}. There are n − 1 numbers in total, and their geometric mean is S^{i-1}. Thus

$$S^{i-1} \le \frac{n-i}{n-1} + \frac{i-1}{n-1} S^{n-1}.$$

Interchanging the two numbers of copies gives

$$S^{n-i} \le \frac{i-1}{n-1} + \frac{n-i}{n-1} S^{n-1}.$$

Add these inequalities:

$$S^{i-1} + S^{n-i} \le 1 + S^{n-1}.$$

At i = 1 or i = n, the same formulas simply give equality. Substituting the bound into the expression for E,

$$E \le \sum_{i=1}^{n} a_i (1 + S^{n-1}) = S(1 + S^{n-1}) = S + S^n.$$

Check that the bound is best possible.

Use one entry $t = S - (n-1)\varepsilon$ and let all the others be ε > 0. If S ≤ 1, put t first; every rearrangement then gives $E \ge t + t^n$. If S > 1, take t > 1 and put it last; every rearrangement again gives $E \ge t^n + t$.

As ε decreases to zero, these lower bounds approach $S + S^n$. Thus a smaller constant cannot work, and

$$c_{\min} = S + S^n.$$