Regional Mathematical Olympiad-2015

Time: 3 hours **December 06, 2015**

Instructions:

- **Calculators (in any form) and protractors are not allowed.**
- **Rulers and compasses are allowed.**
- **Answer all the questions.**
- **All questions carry equal marks. Maximum marks: 102.**
- **Answer to each question should start on a new page. Clearly indicate the question number.**

1. In a cyclic quadrilateral $ABCD$, let the diagonals AC and BD intersect at X. Let the circumcircles of triangles AXD and BXC intersect again at Y. If X is the incentre of triangle ABY, show that $\angle CAD = 90^\circ$.

2. Let $P_1(x) = x^2 + a_1x + b_1$ and $P_2(x) = x^2 + a_2x + b_2$ be two quadratic polynomials with integer coefficients. Suppose $a_1 \neq a_2$ and there exist integers $m \neq n$ such that $P_1(m) = P_2(n)$, $P_2(m) = P_1(n)$. Prove that $a_1 - a_2$ is even.

3. Find all fractions which can be written simultaneously in the forms $\dfrac{7k-5}{5k-3}$ and $\dfrac{6l-1}{4l-3}$, for some integers k, l.

4. Suppose 28 objects are placed along a circle at equal distances. In how many ways can 3 objects be chosen from among them so that no two of the three chosen objects are adjacent nor diametrically opposite?

5. Let ABC be a right triangle with $\angle B = 90^\circ$. Let E and F be respectively the mid-points of AB and AC. Suppose the incentre I of triangle ABC lies on the circumcircle of triangle AEF. Find the ratio BC/AB.

6. Find all real numbers a such that $3 < a < 4$ and $a(a - 3\{a\})$ is an integer. (Here $\{a\}$ denotes the fractional part of a. For example $\{1.5\} = 0.5$; $\{-3.4\}$ $= 0.6$.)

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