# Regional Mathematical Olympiad-2016

**Time: 3 hours** **October 09, 2016**

**Instructions:**

- **Calculators (in any form) and protractors are not allowed.**
- **Rulers and compasses are allowed.**
- **Answer all the questions.**
- **All questions carry equal marks. Maximum marks: 102.**
- **Answer to each question should start on a new page. Clearly indicate the question number.**

1. Let $ABC$ be a right-angled triangle with $\angle B = 90°$. Let $I$ be the incentre of $ABC$. Draw a line perpendicular to $AI$ at $I$. Let it intersect the line $CB$ at $D$. Prove that $CI$ is perpendicular to $AD$ and prove that $ID = \sqrt{b(b-a)}$ where $BC = a$ and $CA = b$.

2. Let $a, b, c$ be positive real numbers such that
$$\frac{a}{1+a} + \frac{b}{1+b} + \frac{c}{1+c} = 1.$$
Prove that $abc \leq 1/8$.

3. For any natural number $n$, expressed in base 10, let $S(n)$ denote the sum of all digits of $n$. Find all natural numbers $n$ such that $n = 2S(n)^2$.

4. Find the number of all 6-digit natural numbers having exactly three odd digits and three even digits.

5. Let $ABC$ be a triangle with centroid $G$. Let the circumcircle of triangle $AGB$ intersect the line $BC$ in $X$ different from $B$; and the circumcircle of triangle $AGC$ intersect the line $BC$ in $Y$ different from $C$. Prove that $G$ is the centroid of triangle $AXY$.

6. Let $\langle a_1, a_2, a_3, \ldots \rangle$ be a strictly increasing sequence of positive integers in an arithmetic progression. Prove that there is an infinite subsequence of the given sequence whose terms are in a geometric progression.

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